DISCHARGE OF TURBULENT JETS INTO A FLUIDIZED BED

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An approximate solution for the discharge of a turbulent jet into a fluidized bed is presented. It is assumed that the boundary layer is divided into two zones. The validity of this model has been confirmed by experiment.

Experiments on special apparatus show that when a horizontal axisymmetric air jet discharges into a fluidized bed at flow velocities exceeding the critical value, a tongue of gas is formed in the bed. In the initial section the tongue expands; then it contracts and closes. The gas tongue is almost horizontal and slightly inclined upward. Its dimensions increase with increasing nozzle exit velocity, decreasing bed bulk-weight, and increasing intensity of fluidization, assuming that the bed particles are the same.

Observation of particle behavior at the boundary of the gas tongue indicates that near the nozzle the particles move toward and then along its surface. The motion of the particles together with the tongue is quickly damped in a direction perpendicular to its surface.

Experimental study indicates that expansion of a turbulent jet in a fluidized bed is accompanied by the formation of a turbulent boundary layer consisting of two zones: a pure gas zone (without solids), and a two-phase zone consisting of gas and particles. The zone boundaries are determined by the gas velocity, which for a monodisperse bed is equal to the critical velocity [1]. This applies to horizontal as well as vertical jets, as confirmed by the experimental data of [2, 3].

The equation of motion for the turbulent two-dimensional boundary layer of a free jet is

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{\partial \overline{u'v'}}{\partial y}=0. \tag{1}$$

According to Prandtl, the apparent turbulent friction [4] is

$$\tau_{xy}=-\rho\,\overline{u'v'} \tag{2}$$

or

$$\tau_{xy}=-\rho\varepsilon_k\frac{\partial \bar{u}}{\partial y}. \tag{3}$$

In accordance with the assumption that there are two zones we will have two values for the apparent turbulent viscosity coefficient ε_{k_1} and ε_{k_2} in each section of the bed. As shown by G. N. Abramovich [5], the general expression for the turbulent viscosity coefficient is

$$\varepsilon_k=\varkappa b\,(u_{\max}-u_{\min}).$$

Then in the gaseous boundary layer

$$\varepsilon_{k_1}=\varkappa_1 b_t\,(u_m-u_s) \tag{4}$$

and in the gas-solid zone of the boundary layer

$$\varepsilon_{k_2}=\varkappa_2(b-b_t)(u_s-u_{g\text{-}s}). \tag{5}$$

In accordance with Prandtl's theory, the fluctuating components normal to the streamline of the averaged motion in each zone of the boundary layer are proportional to the rate of change of jet thickness

$$v_1'\sim\frac{db_t}{d\tau}, \tag{6}$$

$$v_2'\sim\frac{d(b-b_t)}{d\tau}. \tag{7}$$

The increase in the thickness of each zone along the jet can be expressed as follows

$$\frac{db_t}{dx}=\frac{db_t}{d\tau}\left(\frac{d\tau}{dx}\right)_1 \tag{8}$$

and

$$\frac{d(b-b_t)}{dx}=\frac{d(b-b_t)}{d\tau}\left(\frac{d\tau}{dx}\right)_2, \tag{9}$$

where $(dx/d\tau)_1$ and $(dx/d\tau)_2$ are the mean velocities over the section in each zone.

Assuming that the mean velocities in each zone are approximately equal to the arithmetic mean of the velocities at the boundaries of the zones, i.e.,

$$\left(\frac{dx}{d\tau}\right)_1\cong\frac{u_m+u_s}{2} \tag{10}$$

and

$$\left(\frac{dx}{d\tau}\right)_2\cong\frac{u_s+u_{g\text{-}s}}{2}, \tag{11}$$

we obtain the increase in the thickness of the boundary layer by zones

$$\frac{db_t}{dx}\sim\frac{2v_1'}{u_m+u_s} \tag{12}$$

and

$$\frac{d(b-b_t)}{dx}\sim\frac{2v_2'}{u_s+u_{g\text{-}s}}. \tag{13}$$

Assuming that the increase in the thickness of the boundary layer along the jet in each zone is constant, we can write

$$\frac{b_t}{x+x_0}\sim\frac{2v_1'}{u_m+u_s} \tag{14}$$

and

$$\frac{b-b_t}{x+x_0}\sim\frac{2v_2'}{u_s+u_{g\text{-}s}}. \tag{15}$$

Since the velocity gradients over the thickness of the boundary layer in each zone can be approximated by

$$\frac{\partial \bar{u}_1}{\partial y} \cong \frac{u_m - u_s}{b_t} \text{ and } \frac{\partial u_2}{\partial y} \cong \frac{u_s - u_{g\text{-}s}}{b - b_t},$$

the turbulent friction in the zones will be

$$(\tau_{xy})_1 = \varkappa_1 \rho_g (u_m - u_s)^2, \tag{16}$$

$$(\tau_{xy})_2 = \varkappa_2 \rho_f' (u_s - u_{g\text{-}s})^2. \tag{17}$$

On the other hand, assuming u' = v' for each zone, we can write

$$(\tau_{xy})_1 = \rho_g (v_1')^2, \tag{18}$$

$$(\tau_{xy})_2 = \rho_f' (v_2')^2. \tag{19}$$

By comparing Eqs. (16), (17), (18), and (19) we can obtain expressions for v_1' and v_2' and, substituting into Eqs. (14) and (15), the thickness of the boundary layer along the length of the jet in each zone can be determined

$$\frac{b_t}{x + x_0} = C_1 \frac{u_m - u_s}{u_m + u_s} \tag{20}$$

and

$$\frac{b - b_t}{x + x_0} = C_2 \frac{u_s - u_{g\text{-}s}}{u_s + u_{g\text{-}s}}. \tag{21}$$

Here, $C_1 = \varkappa_1 2^{1/2}$, $C_2 = \varkappa_2 2^{1/2}$.

The over-all thickness of the boundary layer is given by.

$$\frac{b}{x + x_0} = C_1 \frac{u_m - u_s}{u_m + u_s} + C_2 \frac{u_s - u_{g\text{-}s}}{u_s + u_{g\text{-}s}} \tag{22}$$

and the change in thickness by

$$\frac{db}{dx} = C_1 \left[x \frac{d}{dx} \left(\frac{u_m - u_s}{u_m + u_s} \right) + \frac{u_m - u_s}{u_m + u_s} \right] + C_2 \frac{u_s - u_{g\text{-}s}}{u_s + u_{g\text{-}s}}. \tag{23}$$

At $x = x_t$, since $u_m = u_s$, the thickness of the boundary layer is given by

$$\frac{b}{x_0 + x_t} = C_2 \frac{u_s - u_{g\text{-}s}}{u_s + u_{g\text{-}s}}. \tag{24}$$

When a liquid jet discharges into a fluidized bed composed of a liquid and solid particles, all the relations derived above remain valid.

Because of the presence of two zones the density of the boundary layer is not uniform over its thickness. In the gas zone the density is equal to the density of the gas and in the case of an isothermal incompressible gas, the flow will be constant. In the "gas-solid" zone the density is hundreds of times higher than the gas density owing to the presence of the particles. In this zone the flow velocity varies from the critical velocity to the fluidization velocity and, consequently, the density changes value. For an exact determination of the mean velocity and mean density it is necessary to know the law of variation of density over the

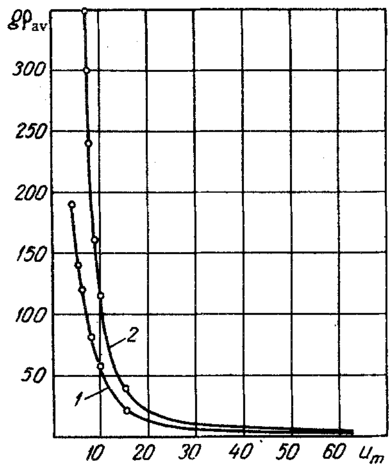

Fig. 1. Average density of jet (kgf/m³) as a function of the gas velocity on the axis (m/sec) at u_s = 10 m/sec; C_1 = = 0.73; C_2 = 0.1: 1) $g\rho_f'$ = 250 kgf/m³; 2) 500.

thickness of the zone. With a certain assumption it is possible to take the density ρ_f' of the "gas-solid" zone equal to 0.5 $(\rho_g + \rho_f)$, and when $\rho_{sol} > \rho_g$ equal to half the density of the fluidized bed (0.5 ρ_f).

The mean density of the boundary layer is obtained averaging the densities of the two zones. Since the thickness of the zones is not the same along the length of the jet, the mean density will also vary. From Eq. (21) it follows that the thickness of the "gas-solid" zone increases continuously, and therefore the mean density also increases.

For each cross section of the boundary layer the momentum can be expressed in terms of the mean density and represented as the sum of the momenta of each zone.

Then for an axisymmetric jet

$$b^2 \rho_{av} u_{av}^2 = b_t^2 \rho_g u_{av_1}^2 + (b^2 - b_t^2) \rho_f' u_{av_2}^2, \tag{25}$$

where

$$u_{av} = \frac{u_m + u_{g\text{-}s}}{2}; \quad u_{av_1} = \frac{u_m + u_s}{2};$$

$$u_{av_2} = \frac{u_s + u_{g\text{-}s}}{2}.$$

Substituting the values of b_t and b from Eqs. (20) and (21) into (25), we obtain an expression for ρ_{av}.

$$\rho_{av} = \rho_g \frac{1}{(1+A)^2} \frac{u_{av_1}^2}{u_{av}^2} + \rho_f' \left[1 - \frac{1}{(1+A)^2} \right] \frac{u_{av_2}^2}{u_{av}^2}, \tag{26}$$

where

$$A = \frac{C_2}{C_1} \frac{(u_s - u_{g\text{-}s})}{(u_s + u_{g\text{-}s})} \frac{(u_m + u_s)}{(u_m - u_s)}.$$

Thus, the average density of the boundary layer depends on the densities of the gas and the fluidized

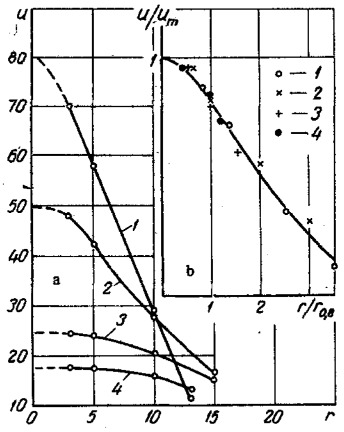

Fig. 2. Velocity profiles in various sections of the jet (a) and dimensionless velocity profile of a horizontal jet (b) at $u_0 = 303$ m/sec and $r_0 = 2$ mm: 1) x = 20 mm 2) 30; 3) 55; 4) 65.

bed, on the critical particle soaring velocity and on the velocity at the jet axis (on the principal section).

For known ρ_g, ρ_f, C_1, C_2, u_s, u_{g-s}, and u_m it is possible to determine the variation of the average density along the length of the principal section of the jet. Fig. 1 presents the average density on the principal section of the jet as a function of the velocity on the jet axis, calculated with Eq. (26) for an air jet at $\rho_g = 0.102$ kg $\cdot$ sec^2 $\cdot$ m^{-4} at coefficient values of $C_1 = 0.73$ and $C_2 = 0.1$.

We see that the density is not constant along the length of the jet, but increases with decrease in the velocity at the jet axis. At the same flow velocity the average density increases together with the density of the fluidized bed.

In the general case the average density in sections of the jet varies from the jet exit density to the density of the fluidized bed. If the jet density is close to the density of the fluidized bed, the average density will vary little along the length of the jet. This may be the case when a liquid jet discharges into a "liquid-solid" fluidized bed or when a gas jet flows into a low-density bed, i.e., a bed with low solid concentration or a low-density solid phase.

To determine the variation in the velocity of a turbulent axisymmetric jet we solve the momentum equations by Abramovich's method for free turbulent jets and jets in parallel flow [5,6].

When a turbulent jet enters a fluidized bed, the integral equation of momentum conservation has the form

$$\int_0^M \rho_{av}(u-u_{g\text{-}s})^2\, 2\pi r dr = \pi \rho_g r_0^2 (u_0-u_{g\text{-}s})^2. \tag{27}$$

Assuming the velocity fields are similar in any section of the jet, we determine the relation between the velocity u and the velocity u_m on the jet axis from Schlichting's equation [7]

$$\frac{u-u_{g\text{-}s}}{u_m-u_{g\text{-}s}} = \left[1-\left(\frac{r}{b}\right)^{1,5}\right]^2. \tag{28}$$

The assumption of similarity for the velocity fields in any section requires experimental verification.

After substituting (28) into (27) we obtain

$$2\rho_{av} b^2 (u_m-u_{g\text{-}s})^2 \int_0^1 \left[1-\left(\frac{r}{b}\right)^{1,5}\right]^4 \frac{r}{b}\, d\frac{r}{b} = $$

$$= \rho_g r_0^2 (u_0-u_{g\text{-}s}), \tag{29}$$

and from (29) the velocity on the axis will be

$$u_m - u_{g\text{-}s} = \frac{r_0(u_0-u_{g\text{-}s})}{0.366\, b}\sqrt{\frac{\rho_g}{\rho_{av}}}. \tag{30}$$

Substituting b from (22) and ρ_{av} from (26), we finally obtain

$$u_m - u_{g\text{-}s} =$$

$$= \frac{(u_0-u_{g\text{-}s})\, r_0}{0.366\left[C_1\dfrac{u_m-u_s}{u_m+u_s} - C_2\dfrac{u_s-u_{g\text{-}s}}{u_s+u_{g\text{-}s}}\right](x+x_0)} \times$$

$$\times \rho_g^{1/2}\left\{\rho_g\left[\frac{1}{1+A}\right]^2\frac{(u_m+u_s)^2}{(u_m+u_{g\text{-}s})^2} + \right.$$

$$\left. + \rho_f'\left[1-\frac{1}{(1+A)^2}\right]\frac{(u_s+u_{g\text{-}s})^2}{(u_m+u_{g\text{-}s})^2}\right\}^{-1/2} \tag{31}$$

Thus, the velocity on the principal section of the jet decreases along its length. The fall in flow veloc-

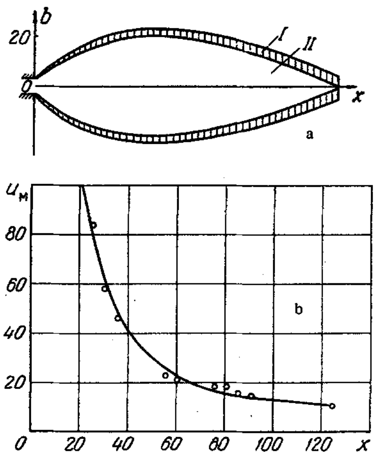

Fig. 3. Variation of velocity on jet axis (a) and calculated profile (b) at $u_0 = 230$ m/sec and $r_0 = 3$ mm; I) "gas-solid" zone of boundary layer; II) gas zone of boundary layer; 0) experimental points (in b, x is in mm, and u_M in m/sec).

ity depends to a considerable extent on the density of the fluidized bed. As the latter increases, the flow velocity decreases, the density of the bed exerting an increasing influence for diminishing velocities. At $u_m = u_s$ the average density becomes equal to half the density of the fluidized bed, and comparing (30) and (24) we obtain an expression for the length of the gas tongue

$$x_t + x_0 = = \frac{r_0(u_0 - u_{g\text{-}s})}{0.366(u_s - u_{g\text{-}s})C_2 \dfrac{(u_s - u_{g\text{-}s})}{(u_s + u_{g\text{-}s})}} \sqrt{\frac{\rho_g}{\rho_f'}}. \quad (32)$$

If the jet enters the fluidized bed horizontally, the velocity vector at the boundary between the "gas-solid" zone and the fluidized bed will be directed vertically; accordingly $u_{g-s} = 0$; if the turbulent jet enters the bed flowing vertically upward, the velocity at the boundary of the "gas-solid" zone will be equal to the constrained critical velocity, i.e., $u_{g-s} = u_{c.s}$.

If the experimental coefficients C_1 and C_2 are known with the approximate equations obtained the configuration of the jet in the fluidized bed can be determined.

Comparison with data on the flow of a semibounded horizontal and unbounded vertical axisymmetric jet confirms the validity of the model for an approximate solution of the problem [2].

The similarity of the velocity fields in the gas tongue assumed in solving the problem was experimentally confirmed. Graphs of the variation of the velocity profiles in various sections along the length of a horizontal and vertical jet are presented in Figs.

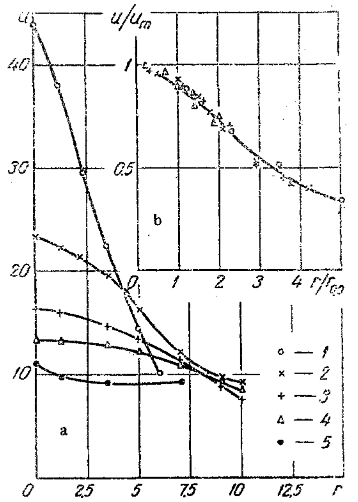

Fig. 4. Velocity profiles in various sections (a) and dimensionless velocity profile of a vertical jet (b) at $u_0 = 190$ m/sec and $r_0 = 2$ mm (u in m/sec; r, x in mm): 1) x = 10 mm; 2) 20; 3) 30; 4) 40; 5) 50.

2 and 4, which also include graphs of the dimensionless velocity profile

$$\frac{u}{u_m} = f\left(\frac{r}{r_{0.8}}\right),$$

where $r_{0.8}$ is the radius of the jet corresponding to a flow velocity $u = 0.8u_m$.

The values of C_1 and C_2 remain practically the same for different exit velocities and initial radii of

Experimental Values of the Coefficients C_1 and C_2

Nozzle diameter, mm	Exit velocity, m/sec	Length of jet, mm	C_1	C_2
For a horizontal jet in a fluidized bed of copolymer particles 4 × 3.72 mm = = 1.17; $\gamma_b = 502$ kg/m^3; $u_s = 10.4$ m/sec.				
4	303.0	118	0.84	0.05
6	134.8	72	0.89	0.05
6	112.4	60	0.83	0.05
6	52.3	28	0.85	0.05
6	110.2	60	0.80	0.05
4	174.0	65	0.81	0.05
4	198.2	73	0.85	0.05
For a vertical jet in a fluidized bed of copolymer particles 3.33 × 3.16 mm; W = 1.10; $\gamma_b = 560$ kg/m^3; $u_s = 8.4$ m/sec.				
4	192.2	58	0.904	0.07
4	137.0	40	0.847	0.07
4	83.4	25	0.889	0.07

a jet flowing into a fluidized bed of constant density (see table).

The relation between C_1 and C_2 can be expressed as

$$C_1 \cong C_2 \sqrt{\frac{\rho_f'}{\rho_g}}.$$

Thus, to calculate the behavior of the jet it is sufficient to know C_1.

Figure 3b presents the calculated and the experimental value of u_m. The agreement is satisfactory. In the calculations we assumed $x_0 = 0$.

The calculated jet configurations (Fig. 3) are similar to those visually observed and show that at $\rho_f \gg \rho_g$ the thickness of the "gas-solid" zone of the boundary layer is small, which is a consequence of momentum transfer between phases of unequal density. Accordingly, the jet contracts. Reduction of the jet cross section is observed at $u_m = 2u_s$.

The configurations obtained confirm the inevitability of a discontinuity when the density of the jet is much less than the density of the fluidized bed under conditions of formation of a vertical gas tongue whose length is less than the thickness of the bed, and of a horizontal tongue within the bed.

NOTATION

u, u_0, u_{max}, u_{min}, u_m, u_s, $u_{c.s}$, u_{g-s} are flow velocities: in any section of the jet, in the initial section, the maximum and minimum sections at edge of boundary layer, at the axis of the axisymmetric jet, the free critical soaring velocity, the constrained critical soaring velocity, at the boundary between the "gas-solid" zone of the boundary layer and the fluidized bed; ε_{k_1}, ε_{k_2} are the turbulent viscosity in the gas and gas-solid zones of the boundary layer; v_1', v_2' are the fluctuating velocity components in the zones of the boundary layer; x and x_0 are the distances: from nozzle outlet, to pole of principal jet section; x_t is the length of the gas tongue; b is the boundary-layer thickness (radius of principal section of axisymmetric jet); b_t is the thickness of the gas zone of the boundary layer (radius of gas tongue); r_0 and r are the initial and variable jet radii; τ_{xy} is the turbulent friction; C_1 and C_2 are experimental coefficients; ρ_{av}, ρ_f, ρ_f', ρ_g, ρ_M are densities: average over jet cross section, of fluidized bed, in "gas-solid" zone, of gas, and of solids; W is the fluidization number; γ_b is the bulk weight of bed.

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